USN

06MAT11

First Semester B.E. Degree Examination, December 2010

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Answer all objective questions only in OMR sheet of the answer booklet.
3. Answer to the objective type questions on sheet other than OMR sheet will not be valued.

PART – A

1 a. Choose the right answer :

i) The n^{th} derivative of log(ax + b) is

A) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ B) $\frac{(-1)^{n-1}n!a^n}{(ax+b)^{n+1}}$ C) $\frac{(-1)^n n!a^n}{(ax+b)^n}$ D) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^{n+1}}$

ii) The angle between the radius vector and the tangent for the curve $r = a$ is

A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) ZERO

iii) $\frac{d^{2n}(x^2-1)^n}{dx^{2n}}$ is

A) 2 (n!) B) $2nx^{2n-1}$ C) (2n)! D) $2nx^{2n-2}$

iv) If A (p, r) is a point on a curve in pedal equation, then p refers to,

A) $\sqrt{x^2+y^2}$

B) $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

C) Perpendicular distance between the point A and the tangent to the curve at A.

D) Perpendicular distance between the origin and the tangent to the curve at point A. **(04 Marks)**

b. Find the n^{th} derivative of $y = \sin(2x+3) + e^{3x} + (5x-3)^{10} + \frac{1}{4x+5}$. **(04 Marks)**

c. If $y = \sin \log(x^3 + 3x^2 + 3x + 1)$, show that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+9)y_n = 0$ **(06 Marks)**

d. Find the angle between the curves $r = a\sec^3\left(\frac{\theta}{3}\right)$ and $r = b\,\mathrm{cosec}^3\left(\frac{\theta}{3}\right)$. **(06 Marks)**

2 a. Choose the right answer :

i) If $u = \sin(x + ay) + \cos(x - ay)$ implies $u_{yy} = a^2 u_{xx}$ then $u = f(x + y) + g(x - y)$ implies

A) $u_{yy} + u_{xx} = 0$ B) $xu_x + yu_y = u$ C) $xu_x + yu_y = -u$ D) $u_{yy} = u_{xx}$

ii) If $u = \sin\left(\frac{y}{x}\right) + \tan\left(\frac{x}{y}\right)$ then u is homogeneous function of order,

A) -1 B) 1 C) ZERO D) None of these

iii) If $J\left(\frac{u,v}{x,y}\right) \neq 0$ then

A) Only x, y are independent B) x, y are independent and u, v are independent
C) Only u, v are independent D) Cannot predict

iv) If 20% error is made in each of the independent variables then the percentage error in w, if w = xyzuv is

A) 100% B) 20% C) $(20)^5$% D) 5% **(04 Marks)**

b. A balloon is in the form of the right circular cylinder of radius 1.5 cms and length 4 cms and is surmounted by hemispherical ends. If the radius is increased by 0.01 cms and the length is increased by 0.05 cms, find the percentage change in the volume of the balloon. **(04 Marks)**

c. If $u = (x^2 + y^2 + z^2)^{-1/2}$ then find i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$; ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. **(06 Marks)**

d. i) If $u = x^2 - y^2, v = 2xy$, find $J\left(\frac{u,v}{x,y}\right)$; ii) If $x^2 + xy + y^3 = 2$, find $\frac{d[x^3y]}{dx}$. **(06 Marks)**

3 a. Choose the right answer :

i) The value of $\int_0^{\pi/2} \mathrm{Cos}^3\left(\frac{\theta}{2}\right) d\theta$ is

A) $\frac{4}{3}$ B) $\frac{2}{3}$ C) $\frac{2}{3}\pi$ D) $\frac{4}{3}\pi$

ii) The curve $x^{2/5} + y^{2/7} = a$ is symmetric

A) Only about x – axis B) About both x and y axes

C) Only about y – axis D) About the line y = x

iii) If $I_1 = \int_0^{\pi/4} \tan^6 x dx$ and $I_2 = \int_{\pi/4}^{\pi/2} \cot^6 x dx$ then

A) $I_1 = I_2 + \pi/4$ B) $I_1 = I_2 - \pi/4$ C) $I_1 = I_2$ D) $I_1 = 2I_2$

iv) The reduction formula of $\int \sec^n x\, dx$ is

A) $\frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ B) $\frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-1}{n-2} I_{n-2}$

C) $\frac{\tan x \sec^{n-2} x}{n-2} + \frac{n-1}{n-2} I_{n-2}$ D) $\frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^n x\, dx$. **(04 Marks)**

c. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^7} dx$. **(06 Marks)**

d. Trace the curve $3ax^2 = y(y-a)^2$. **(06 Marks)**

4 a. Choose the right answer :

i) The perimeter of the curve r = a is

A) 4a B) πa C) 2a D) $2\pi a$

ii) If v_1 and v_2 are volumes of the solids of revolution got by rotating respectively, 'the parabola $y^2 = 4ax$, above x –axis, between x = 0 to x = 2a' and 'the same parabola $y^2 = 4ax$ both above and below x – axis between x = 0 to x = 2a' then

A) $v_2 = 2v_1$ B) $v_2 = 4v_1$ C) $v_2 = v_1$ D) None of these

iii) If the axis of the revolution is the y – axis then the surface area of revolution is

A) $\int_{x_1}^{x_2} 2\pi y ds$ B) $\int_{y_1}^{y_2} 2\pi x ds$ C) $\int_{y_1}^{y_2} 2\pi\sqrt{1+\left(\frac{dy}{dx}\right)^2} dy$ D) $\int_{x_1}^{x_2} 2\pi\sqrt{1+\left(\frac{dx}{dy}\right)^2} dx$

iv) The area bounded by the curve in polar form is

A) $\int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$ B) $\int_{r_1}^{r_2} \frac{1}{2}\theta^2 dr$ C) $\int_{\theta_1}^{\theta_2} \sqrt{1+\left(\frac{dr}{d\theta}\right)^2} d\theta$ D) $\int_{x_1}^{x_2} y dx$ **(04 Marks)**

b. Compute the total arc – length (perimetre) of the cardiod $r = 2(1 + \cos\theta)$. **(04 Marks)**

c. Find the area enclosed between the cycloid $x = a(t - \mathrm{Sin}\, t), y = a(1 - \cos t)$ and its base. **(06 Marks)**

d. Find the volume of the solid generated by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x – axis. **(06 Marks)**

PART – B

5 a. Choose the right answer :

i) The order of the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-8y=0$ is :

A) 2 B) ZERO C) 1 D) 3

ii) The integrating factor of $\frac{dy}{dx}+P(x)y=Q(x)$ is

A) Only function of y B) Only function of x
C) Function of x and y D) Function of dy/dx

iii) The differential equation $\frac{dy}{dx}+\frac{y}{x}=0$ can be solved

A) Only by variable separable, or exact method
B) Only by homogeneous or linear d.c. method
C) By all the methods mentioned in (A) and (B)
D) Only by variable separable method.

iv) The differential equation $(x-2)dy=(2-y)dx$ is

A) Only R.H.S. exact B) Only L.H.S. exact
C) Not exact D) Exact d.e. **(04 Marks)**

b. Solve $\text{Cos}y\frac{dy}{dx}-\text{Sin}y\frac{1}{1+x}=(x+1)^2$. **(04 Marks)**

c. Solve $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xy\,e^{xy^2}-3y^2\right)dy=0$. **(06 Marks)**

d. Find the orthogonal trajectory of $r = a(1 + \text{Cos}\,\theta)$. **(06 Marks)**

6 a. Choose the right answer :

i) For a series of positive terms $\sum_{n=1}^{\infty}u_n$ if $\underset{n\to\infty}{\text{Lt}}(u_n)$ does not tend to zero then the series is,

A) Convergent B) Cannot conclude C) Oscillatory D) Divergent

ii) If positive term series $\sum_{n=1}^{\infty}u_n$, $\sum_{n=1}^{\infty}v_n$ both are divergent then $\sum_{n=1}^{\infty}u_n$ - $\sum_{n=1}^{\infty}v_n$ is

A) Divergent B) Convergent C) Cannot predict D) Oscillatory

iii) The series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots.$ is

A) Divergent B) Convergent C) Oscillatory D) None of these

iv) If an infinite series $\sum u_n$ is convergent and if $\underset{n\to\infty}{\text{Lt}}\frac{u_n}{u_{n+1}}=k_1$, $\underset{n\to\infty}{\text{Lt}}\frac{u_{n+1}}{u_{n+2}}=k_2$ then,

A) $k_1 = k_2$ B) $k_1 \neq k_2$ C) $k_1 < k_2$ D) $k_1 > k_2$ **(04 Marks)**

b. Test the convergence of $\frac{9}{6.7.8}+\frac{11}{11.12.13}+\frac{13}{16.17.18}+\frac{15}{21.22.23}+\ldots\ldots\ldots$ **(04 Marks)**

c. Test the convergence of $1+2+3+4+5+\frac{3^2}{4^2}x+\frac{3^2.4^2}{4^2.5^2}x^2+\frac{3^2.4^2.5^2}{4^2.5^2.6^2}x^3+\ldots\ldots\ldots.$ **(06 Marks)**

d. i) Using Leibinitz's test, detect the nature of the series, $\frac{2}{3}-\frac{3}{4}+\frac{4}{5}-\frac{5}{6}+\ldots\ldots$

ii) Define conditional convergence and give one example. **(06 Marks)**

7 a. Choose the right answer :

i) The line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-5}{11}$ and the plane $5x + 4y - 2z = 8$,

A) Intersect at an angle $\pi/4$ B) Intersect at an angle $\pi/6$

C) Are perpendicular D) Are parallel

ii) The direction cosines of x, y, z axes are respectively

A) (-1, 1, 1) (1, -1, 1) (1, 1, -1) B) $\left(\frac{1}{\sqrt{2}},0,0\right)\left(0,\frac{1}{\sqrt{2}},0\right)\left(0,0\frac{1}{\sqrt{2}}\right)$

C) (1, 0, 0) (0, 1, 0) (0, 0, 1) D) (1, -1, -1) (-1, 1, -1) (-1, -1, 1)

iii) A point on a line $\frac{x}{2}=\frac{y+3}{6}=\frac{z-1}{10}$ is

A) (-1, 0, 6) B) (1, 0, 6) C) (-1, 0, -6) D) (1, 0, -6)

iv) A line perpendicular to plane is

A) Perpendicular to all the lines in the plane

B) Perpendicular to one set of parallel lines in the plane

C) Perpendicular to exactly one line in the plane

D) A line can not be perpendicular to the plane **(04 Marks)**

b. Find the angle between the lines AB and CD, where :
A = (1, 2, 3), B = (4, 5, 9), C = (2, 4, 6), D = (5, 7, 8). **(04 Marks)**

c. Show that the points (1, 1, 1) (2, -3, 11) (4, -2, 4) (1, 0, 4) are co-planar. Find the equation of the plane passing through the given points. **(06 Marks)**

d. Find the shortest distance between the straight lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$$

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$ **(06 Marks)**

8 a. Choose the right answer :

i) If $\vec{r}=\overrightarrow{op}$ with p = (x, y, z), 0 = (0, 0, 0), $x = t^2$, $y = 2t - 3$, $z = 3t - 5$ then $\vec{r}$ at t = 1 is

A) $i - j - 2k$ B) $i + 2j + 3k$ C) $i + j + k$ D) $i - j + 2k$

ii) Recognize the meaningless expression, for $\vec{F}$ = vector function and ϕ = scalar function

A) grad $(\text{div}\,\vec{F})$ B) grad (grad ϕ) C) curl $(\text{grad}\,\vec{F})$ D) div (grad ϕ)

iii) Curl $(\text{curl}\,\vec{F})$ is

A) grad $(\text{div}\,\vec{F})$ B) $\nabla^2 F$ C) grad $(\text{div}\,\vec{F})$- $\nabla^2 F$ D) $(\text{curl})^2\,\vec{F}$

iv) If $\vec{F}$ is a vector point function, recognize the true statement :

A) $\nabla \times \vec{F} = -\vec{F} \times \nabla$ B) $\nabla \cdot \vec{F} = \vec{F} \cdot \nabla$ C) $\nabla \cdot \nabla\vec{F} = \nabla \times \nabla\vec{F}$ D) $\nabla \cdot \vec{F} \neq \vec{F} \cdot \nabla$ **(04 Marks)**

b. Find the angle between the normals to the surface xy = z2 at the points (4, 1, 2) and (3, 3, -3). **(04 Marks)**

c. i) For what value of 'a', vector point function $\vec{F}$ is solenoidal
if $\vec{F} = (2x + 3y)\, i - (3x + 4y)\, j + (y - az)\, k$

ii) Is $\vec{F} = (6xy + z^3)\, i + (3x^2 - z)\, j + (3xz^2 - y)\, k$, irrotational? **(06 Marks)**

d. If $\vec{F}$ and ϕ are vector and scalar point functions respectively then prove that
$\text{div}\,(\phi\vec{F}) = \phi\,(\text{div}\,\vec{F}) + (\text{grad}\,\phi)\cdot\vec{F}$. **(06 Marks)**

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10MAT11

First Semester B.E. Degree Examination, January 2011

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answer :

i) If f(x) is continuous in [a, b], differentiable in (a, b) and f(a) = f(b), then there exists ____ $C \in (a, b)$ such that $f'(c) = 0$.
A) unique B) infinite C) al least one D) no such

ii) The Maclaurin's series of f(x) = k(constant) is,
A) f(x) = k B) f(x) = 0 C) does not exist D) f(x) = k!

iii) The n^{th} derivative of $\frac{1}{(x+2)^3}$ is
A) $\frac{(-1)^n (n+2)!}{2!(x+2)^{n+3}}$ B) $\frac{1}{(x+2)^{n+3}}$ C) ZERO D) None of these.

iv) The 12^{th} derivative of $y = e^{\sqrt{2}x} \sin\sqrt{2}x$ is
A) (64)y B) -4096y C) (32)y D) None of these.
(04 Marks)

b. If x = tan(log y), prove that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$ **(06 Marks)**

c. Expand log(sec x) by using the Maclaurin's series expansion up to the term containing x^4. **(05 Marks)**

d. State and prove the Lagrange's mean value theorem. **(05 Marks)**

2 a. Choose the correct answer :

i) Which statement is true?
A) $\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0\times\infty$ are not indeterminate B) 0^0, ∞^0 are not indeterminate
C) 1^∞ is not indeterminate D) None of these.

ii) The angle between r = asinθ and r = bcosθ, is
A) π/2 B) π C) - π/2 D) None of these.

iii) The radius of a curvature in the polar form is,
A) $\frac{[r^2+r_1^2]^{3/2}}{r^2+2r_1^2-rr_2}$ B) $\frac{[r_1^2+r^2]^{3/2}}{r_1^2+2r^2-rr_2}$ C) $\frac{[r^2+r_1^2]^{3/2}}{r^2+2r_1r_2-rr_2}$ D) None of these.

iv) $\lim_{x\to 0} \frac{2^x-3^x}{5^x-6^x}$ is ,
A) $\frac{\log(2/3)}{\log(5/6)}$ B) $\log\left[\frac{2}{3}-\frac{5}{6}\right]$ C) $\log\left[\frac{2/3}{5/6}\right]$ D) None of these.
(04 Marks)

b. Evaluate : i) $\lim_{x\to 0} \frac{\sin x \sin^{-1} x}{x^2}$ ii) $\lim_{x\to 0} \left(\frac{2^x+3^x+4^x}{3}\right)^{1/x}$ **(06 Marks)**

c. Derive an expression for the radius of curvature in the pedal form. **(05 Marks)**

d. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts x-axis. **(05 Marks)**

3 a. Choose the correct answer :

i) If $u = ax^2 + by^2 + abxy$, then $\frac{\partial^3 u}{\partial x^2 \partial y}$ is

A) Zero B) a + b + ab C) ab D) None of these.

ii) The Taylor's series of f(x, y) = xy at (1, 1) is

A) 1 + [(x – 1) + (y – 1)] B) 1 + [(x – 1) + (y – 1)] + [(x – 1)(y – 1)]

C) (x – 1)(y – 1) D) None of these.

iii) The Jacobian of transformation from the Cartesian to polar coordinate system is,

A) r^2 B) $r^2\cos\theta$ C) $r^2\sin\theta$ D) None of these.

iv) If $u = f(x, y)$, $x = \phi(t)$, $y = \psi(t)$, then du/dt is,

A) $\frac{du}{dx}\frac{dx}{dt}+\frac{du}{dy}\frac{dy}{dt}$ B) $\frac{dx}{dt}+\frac{dy}{dt}$ C) $\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{dy}{dt}$ D) None of these.

(04 Marks)

b. If $\sin u = \frac{x^2y^2}{x+y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\tan u$ **(06 Marks)**

c. If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$ and $w = \frac{xz}{y}$, find $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

d. If the H.P. required by the steamer varies as the cube of the velocity and the square of the length, find the percentage change in H.P. for 3% and 4% increase in velocity and length respectively. **(05 Marks)**

4 a. Choose the correct answer :

i) The gradient, divergence, curl are respectively

A) scalar, scalar, vector B) vector, scalar, vector

C) scalar, vector, vector D) vector, vector, scalar

ii) $\vec{V} = y^2z\,\vec{i} + z^2x\,\vec{j} + x^2y\,\vec{k}$ is

A) constant vector B) solenoidal vector C) scalar D) None of these.

iii) Curl grad f is,

A) grad curl f B) curl grad f + grad curl f C) zero D) does not exist.

iv) If the curvilinear system is spherical polar coordinate system then the radius vector R is

A) $r\sin\theta\cos\phi\vec{i}+r\sin\theta\sin\phi\vec{j}+r\cos\theta\vec{k}$ B) $r\sin\theta\vec{i}+r\cos\theta\vec{j}+r\vec{k}$

C) $\vec{i}+\vec{j}+\vec{k}$ D) None of these. **(04 Marks)**

b. If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$, then find grad$\phi$, div$\vec{F}$, curl$\vec{F}$. **(06 Marks)**

c. Prove that div Curl F = $\nabla\cdot\nabla\times F = 0$. **(05 Marks)**

d. Prove that the cylindrical coordinate system is orthogonal. **(05 Marks)**

PART – B

5 a. Choose the correct answer :

i) The value of $\int_0^{\pi}\sin^5 x\cos^6 x\,dx$ is

A) $\frac{5\times3\times1}{11\times9\times7}$ B) $\frac{4\times2}{11\times9}\frac{\pi}{2}$ C) $\frac{2\times4\times2}{11\times9\times7}$ D) None of these.

ii) $x^2 + y^2 = x^2y^2$ is symmetric about

A) x-axis B) y-axis C) the line y = x D) All of these

iii) Surface area of a solid of revolution of the curve y = f(x), if rotated about x-axis, is:

A) $\int_{x=a}^{b}2\pi y\,dx$ B) $\int_{x=a}^{b}2\pi x\,dy$ C) $\int_{x=a}^{b}2\pi y\,ds$ D) $\int_{x=a}^{b}2\pi x\,ds$

iv) Asymptote to the curve $y^2(a-x) = x^3$ is
A) $y = 0$ B) $x = 0$ C) $x = a$ D) None of these.
(04 Marks)

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$. **(06 Marks)**

c. Derive the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. **(05 Marks)**

d. Compute the perimeter of the cardiod $r = a(1 + \cos\theta)$. **(05 Marks)**

6 a. Choose the correct answer :

i) For the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^6 + y = x^4$, the order and degree respectively are
A) 2, 6 B) 3, 2 C) 2, 4 D) None of these.

ii) $\frac{dy}{dx} + \frac{y}{x} = 0$ is
A) Variable separable and homogeneous B) Linear
C) Homogeneous and exact D) All of these.

iii) $ydx - xdy = 0$ can be reduced to exact, if divided by
A) $x^2 + y^2$ B) y^2 C) xy D) All of these.

iv) Orthogonal trajectory of $y^2 = 4a(x + a)$ is
A) $x^2 = 4a(y + a)$ B) $x^2 + y^2 = a^2$ C) Self orthogonal D) None of these.
(04 Marks)

b. Solve: $(1 + y^2)dx + (x - e^{-\tan^{-1}y})dy = 0$ **(06 Marks)**

c. Solve: $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ **(05 Marks)**

d. Find the orthogonal trajectory of the cardiods $r = a(1 - \cos\theta)$, using the differential equation method. **(05 Marks)**

7 a. Choose the correct answer :

i) Which of the following is not an elementary transformation?
A) Adding two rows B) Adding two columns
C) Multiplying a row by a non-zero number D) Squaring all the elements of the matrix.

ii) Rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is
A) 3 B) 1 C) 2 D) None of these.

iii) The solution of the simultaneous equations $x + y = 0$, $x - 2y = 0$ is
A) only trivial B) only unique C) unique and trivial D) None of these.

iv) Which of the following is in the normal form?

A) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ D) All of these.

(04 Marks)

b. Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$. **(06 Marks)**

c. For what values of λ and μ , the following simultaneous equations have i) No solution ii) a unique solution iii) an infinite number of solutions?
$x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$. **(05 Marks)**

d. Solve, using the Gauss-Jordan method.
$x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$. **(05 Marks)**

8 a. Choose the correct answer :
i) The eigen values of the matrix A exist, if
A) A is a square matrix B) A is singular matrix
C) A is any matrix D) A is a null matrix.
ii) A square matrix A of order 'n' is similar to a square matrix B of the order 'n' if
A) $A = P^{-1}BP$ B) AB = Null matrix C) AB = Unit matrix D) None of these.
iii) Which of these is in quadratic form?
A) $x^2 + y^2 + z^2 - 2xy + yz - zx$ B) $x^3 + y^3 + z^2$
C) $(x - y + z)^2$ D) None of these.
iv) Quadratic form (X'AX) is positive definite, if
A) All the eigen values of A are > 0 B) At least one eigen value of A is > 0
C) All eigen values ≥ 0 and at least one eigen value = 0 D) No such condition.
(04 Marks)

b. Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix
$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **(06 Marks)**

c. If $P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is a modal matrix of the matrix A in Q.No.8(b), and inverse of P is
$P^{-1} = \begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, then transform A in to diagonal form and hence find A^4. **(05 Marks)**

d. Find the nature of the quadratic forms for which corresponding eigen values of the corresponding matrices are given as

Matrix	Eigen values
A	2, 3, 4
B	–3, –4, –5
C	0, 3, 6
D	0, –3, –4
E	–2, 3, 4

(05 Marks)

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